How To: Identify Early Math Difficulties in the Primary Grades

In the early elementary grades, students' success in mathematics can be predicted by assessing their acquisition and use of foundation numeracy skills (Gersten, Jordan, & Flojo, 2005). The term *number sense* is often used as short-hand to describe a child's emerging grasp of fundamental mathematical concepts such as what numbers mean, how sets of objects can be described in numerical terms, counting, and simple operations of mental arithmetic (Chard et al, 2005). *Number sense* is difficult to define with precision because the descriptor encompasses a wide range of early math skills (Clarke & Shinn, 2004). By the time a student has entered kindergarten or 1st grade, however, this term can be framed more concretely as a student's ability to access and use a mental number-line.

In the primary grades, the Common Core State Standards in Mathematics are built on the assumption that the successful math student can rapidly access a mental number line for use in such applied mathematical tasks as counting, making accurate comparisons between number, and estimating amounts. For example, a Kindergarten Counting & Cardinality standard (CCSM.K.CC.2) states that a student will "count forward beginning from a given number within the known sequence (instead of having to begin at 1)." (National Governors Association Center for Best Practices et al., 2010; p. 11). Similarly, a Grade 1 standard for Number & Operations in Base 10 (CCSM.1.NBT.1) sets as a student goal to "count to 120, starting at any number less than 120. " (National Governors Association Center for Best Practices et al., 2010; p. 15). Clearly, these and other math standards for the early grades must depend on students' ability to envision and mentally manipulate an internal number-line.

Early Math Fluency Measures: What They Are. Teachers at the primary level have a pressing need for screening tools that can quickly identify those students who require additional instructional support to address deficient number-sense skills. Early Math Fluency measures are one useful means to assess the strength of a young student's 'number sense' (Chard, et al., 2005) and serve as good predictors of mathematical readiness at Kindergarten and Grade 1. Early Math Fluency measures are examples of Curriculum-Based Measurement (Hosp, Hosp, & Howell, 2007) and include Quantity Discrimination, Missing Number, and Number Identification. All Early Math Fluency assessments have an administration time of 1 minute. Here are brief descriptions for three of these measures:

- *Quantity Discrimination:* The student is presented with pairs of numbers randomly sampled from 1-20 and must identify the larger number in each pair.

- *Missing Number:* The student is presented with response items consisting of 3 sequential numbers with one of those numbers randomly left blank. (Each 3-number series is randomly generated from the pool of numbers 1-20.) The student attempts to name the missing number in each series.

- *Number Identification*: The student is presented with a randomly generated series of numbers ranging from 1-20 and names as many of those numbers aloud as time allows.

Early Math Fluency Measures: How to Access Resources. Teachers who would like to screen their Kindergarten and Grade 1 students for possible number-sense delays can obtain these free Early Math Fluency assessment resources: (1) materials for assessment, (2) guidelines for administration and scoring, and (3) research-based norms.

- *Materials for assessment*. Schools can create their own CBM Early Math Fluency assessment materials at no cost, using NumberFly, a free online application:
  http://www.interventioncentral.org/tools/early-math-fluency-generator

- *Guidelines for administration and scoring.* The following sets of instructions for preparing, administering, and scoring Early Math Fluency assessments appear later in this document:
    - *Early Math Fluency/Quantity Discrimination: Guidelines for Use*
    - *Math Fluency/Missing Number: Guidelines for Use*
    - *Math Fluency/Number Identification: Guidelines for Use*

- *Research-based norms.* A table, *Curriculum-Based Measurement: Early Mathematics Fluency Norms*, is included in this document. These fluency benchmarks were researched by Chard et al. (2005) and provide Fall/Winter/Spring screening norms for Quantity Discrimination, Missing Number, and Number Identification.

References

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment for Effective Intervention, 30*(3), 3-14.

Clarke, B., & Shinn, M. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*, 234–248.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*, 293-304.

Hosp, M.K., Hosp, J. L., & Howell, K. W. (2007). *The ABCs of CBM.* New York: Guilford.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors. Retrieved from http://www.corestandards.org/

Early Math Fluency/Quantity Discrimination: Guidelines for Use

This introduction to the Quantity Discrimination probe provides information about the preparation, administration, and scoring of this Early Math CBM measure. Additionally, it offers brief guidelines for integrating this assessment into a school-wide 'Response-to-Intervention' model.

Quantity Discrimination: Description (Clarke & Shinn, 2004; Gersten, Jordan & Flojo, 2005)

The student is given a sheet containing pairs of numbers. In each number pair, one number is larger than the other. The numbers in each pair are selected from within a predefined range (e.g., no lower than 1 and no higher than 20). During a one-minute timed assessment, the student identifies the larger number in each pair, completing as many items as possible while the examiner records any Quantity Discrimination errors.

Quantity Discrimination: Preparation

The following materials are needed to administer Quantity Discrimination (QD) Early Math CBM probes:

- Student and examiner copies of a QD assessment probe. (Note: Customized QD probes can be created conveniently and at no cost using Numberfly, a web-based application. Visit Numberfly at *http://www.interventioncentral.org/php/numberfly/numberfly.php*).
- A pencil, pen, or marker
- A stopwatch

Quantity Discrimination: Directions for Administration

1. The examiner sits with the student in a quiet area without distractions. The examiner sits at a table across from the student.
2. The examiner says to the student:

   "The sheet on your desk has pairs of numbers. In each set, one number is bigger than the other."

   "When I say, 'start,' tell me the name of the number that is larger in each pair. Start at the top of this page and work across the page [demonstrate by pointing]. *Try to figure out the larger number for each example.. When you come to the end of a row, go to the next row. Are there any questions?* [Pause] *Start. "*

   NOTE: If the student has difficulties with speech production, the examiner can use this alternate wording for directions: *"When I say, 'start,' point to the number that is larger in each pair"*
3. The examiner begins the stopwatch when the student responds aloud to the first item. If the student hesitates on a number for 3 seconds or longer on a Quantity Discrimination item, the examiner says, *"Go to the next one."* (If necessary, the examiner points to the next number as a student prompt.)
4. The examiner marks each Quantity Discrimination error by marking a slash (/) through the incorrect response item on the examiner form.
5. At the end of one minute, the examiner says, *"Stop"* and writes in a right-bracket symbol ( ]) on the examiner form after the last item that the student had attempted when the time expired. The examiner then collects the student Quantity Discrimination sheet.

Quantity Discrimination: Scoring Guidelines

Correct QD responses include:

- Quantity Discriminations read correctly
- Quantity Discriminations read incorrectly but corrected by the student within 3 seconds

Incorrect QD responses include:

- The student's reading the smaller number in the QD number pair
- Correct QD responses given after hesitations of 3 seconds or longer
- The student's calling out a number other than appears in the QD number pair
- Response items skipped by the student

To calculate a Quantity Discrimination fluency score, the examiner:

1. counts up all QD items that the student attempted to answer and
2. subtracts the number of QD errors from the total number attempted.
3. The resulting figure is the number of correct Quantity Discrimination items completed.(QD fluency score).

Quantity Discrimination Probes as Part of a Response to Intervention Model

- Universal Screening: To proactively identify children who may have deficiencies in development of foundation math concepts, or 'number sense' (Berch, 2005), schools may choose to screen all kindergarten and first grade students using Quantity Discrimination probes. Those screenings would take place in fall, winter, and spring. Students who fall below the 'cutpoint' of the 35th percentile (e.g., Gersten, Jordan & Flojo, 2005).of the grade norms on the QD task would be identified as having moderate deficiencies and given additional interventions to build their 'number sense' skills.

- Tier I (Classroom-Based) Interventions: Teachers can create Quantity Discrimination probes and use them independently to track the progress of students who show modest delays in their math foundation skills.

- Tier II (Individualized) Interventions. Students with more extreme academic delays may be referred to a school-based problem-solving team, which will develop more intensive, specialized interventions to target the student's academic deficits (Wright, 2007). Quantity Discrimination probes can be used as one formative measure to track student progress with Tier II interventions to build foundation math skills.

Quantity Discrimination: Measurement Statistics

Test-Retest Reliability Correlations for Quantity Discrimination Probes		
Time Span	*Correlation*	*Reference*
13-week interval	0.85	Clarke & Shinn (2004)
26-week interval	0.86	Clarke & Shinn (2004)

Predictive Validity Correlations for Quantity Discrimination Probes		
Predictive Validity Measure	*Correlation*	*Reference*
Curriculum-Based Measurement Math Computation Fluency Probes: Grade 1 Addition & Subtraction (Fall Administration of QD Probe and Spring Administration of Math Computation Probe)	0.67	Clarke & Shinn (2004)

Woodcock-Johnson Tests of Achievement: Applied Problems subtest (Fall Administration of QD Probe and Spring Administration of WJ-ACH subtest)	0.79	Clarke & Shinn (2004)
Number Knowledge Test	0.53	Chard, Clarke, Baker, Otterstedt, Braun & Katz.(2005) cited in Gersten, Jordan & Flojo (2005)

References

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment For Effective Intervention, 30*(2), 3-14.

Clarke, B., & Shinn, M. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*, 234–248.

Gersten, R., Jordan, N.C., & Flojo, J.R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38,* 293-304.

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38,* 333-339..

Wright, J. (2007). *The RTI toolkit: A practical guide for schools*. Port Chester, NY: National Professional Resources, Inc.

Early Math Fluency/Missing Number: Guidelines for Use

This introduction to the Missing Number probe provides information about the preparation, administration, and scoring of this Early Math CBM measure. Additionally, it offers brief guidelines for integrating this assessment into a school-wide ‘Response-to-Intervention’ model.

Missing Number: Description (Clarke & Shinn, 2004; Gersten, Jordan & Flojo, 2005)
The student is given a sheet containing multiple number series. Each series consists of 3-4 numbers that appear in sequential order. The numbers in each short series are selected to fall within a predefined range (e.g., no lower than 1 and no higher than 20). In each series, one number is left blank (e.g., ‘1 2 _ 4’).During a one-minute timed assessment, the student states aloud the missing number in as many response items as possible while the examiner records any Missing Number errors.

Missing Number: Preparation
The following materials are needed to administer Missing Number (MN) Early Math CBM probes:

- Student and examiner copies of a MN assessment probe. (**Note:** Customized MN probes can be created conveniently and at no cost using Numberfly, a web-based application. Visit Numberfly at *http://www.interventioncentral.org/php/numberfly/numberfly.php*).
- A pencil, pen, or marker
- A stopwatch

Missing Number: Directions for Administration

6. The examiner sits with the student in a quiet area without distractions. The examiner sits at a table across from the student.

7. The examiner says to the student:

   “The sheet on your desk has sets of numbers. In each set, a number is missing.”

   “When I say, 'start,' tell me the name of the number that is missing from each set of numbers. Start at the top of this page and work across the page [demonstrate by pointing]. *Try to figure out the missing number for each example.. When you come to the end of a row, go to the next row. Are there any questions?* [Pause] *Start. “*

   NOTE: If the student has difficulties with speech production, the examiner can give the student a pencil and use this alternate wording for directions: *“When I say, 'start, write in the number that is missing from each set of numbers.”*

8. The examiner begins the stopwatch when the student reads the first number aloud. If the student hesitates on a number for 3 seconds or longer on a Missing Number item, the examiner says the correct number aloud and says, *“Go to the next one.”* (If necessary, the examiner points to the next number as a student prompt.)

9. The examiner marks each Missing Number error by marking a slash (/) through the incorrect response item on the examiner form.

10. At the end of one minute, the examiner says, *"Stop"* and writes in a right-bracket symbol ( ]) on the examiner form after the last item that the student had attempted when the time expired. The examiner then collects the student Missing Number sheet.

Missing Number: Scoring Guidelines

Correct MN responses include:

- Missing numbers read correctly
- Missing numbers read incorrectly but corrected by the student within 3 seconds

Incorrect MN responses include:

- Missing numbers read incorrectly
- Missing numbers read correctly after hesitations of 3 seconds or longer
- Response items skipped by the student

To calculate a Missing Number fluency score, the examiner:

1. counts up all MN items that the student attempted to read aloud and
2. subtracts the number of MN errors from the total number attempted.
3. The resulting figure is the number of correct Missing Number items completed.(MN fluency score).

Missing Number Probes as Part of a Response to Intervention Model

- Universal Screening: To proactively identify children who may have deficiencies in development of foundation math concepts, or 'number sense' (Berch, 2005), schools may choose to screen all kindergarten and first grade students using Missing Number probes. Those screenings would take place in fall, winter, and spring. Students who fall below the 'cutpoint' of the 35^{th} percentile (e.g., Gersten, Jordan & Flojo, 2005).of the grade norms on the MN task would be identified as having moderate deficiencies and given additional interventions to build their 'number sense' skills.

- Tier I (Classroom-Based) Interventions: Teachers can create Missing Number probes and use them independently to track the progress of students who show modest delays in their math foundation skills.

- Tier II (Individualized) Interventions. Students with more extreme academic delays may be referred to a school-based problem-solving team, which will develop more intensive, specialized interventions to target the student's academic deficits (Wright, 2007). Missing Number probes can be used as one formative measure to track student progress with Tier II interventions to build foundation math skills.

Missing Number: Measurement Statistics

Test-Retest Reliability Correlations for Missing Number Probes		
Time Span	*Correlation*	*Reference*
13-week interval	0.79	Clarke & Shinn (2004)
26-week interval	0.81	Clarke & Shinn (2004)

Predictive Validity Correlations for Missing Number Probes		
Predictive Validity Measure	*Correlation*	*Reference*
Curriculum-Based Measurement Math Computation Fluency Probes: Grade 1 Addition & Subtraction (Fall Administration of MN Probe and Spring Administration of Math Computation Probe)	0.67	Clarke & Shinn (2004)

Woodcock-Johnson Tests of Achievement: Applied Problems subtest (Fall Administration of MNF Probe and Spring Administration of WJ-ACH subtest)	0.72	Clarke & Shinn (2004)
Number Knowledge Test	0.61	Chard, Clarke, Baker, Otterstedt, Braun & Katz.(2005) cited in Gersten, Jordan & Flojo (2005)

References

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment For Effective Intervention, 30*(2), 3-14.

Clarke, B., & Shinn, M. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*, 234–248.

Gersten, R., Jordan, N.C., & Flojo, J.R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38,* 293-304.

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38,* 333-339..

Wright, J. (2007). *The RTI toolkit: A practical guide for schools*. Port Chester, NY: National Professional Resources, Inc.

Early Math Fluency/Number Identification: Guidelines for Use

This introduction to the Number Identification probe provides information about the preparation, administration, and scoring of this Early Math CBM measure. Additionally, it offers brief guidelines for integrating this assessment into a school-wide 'Response-to-Intervention' model.

Number Identification: Description (Clarke & Shinn, 2004; Gersten, Jordan & Flojo, 2005)
The student is given a sheet containing rows of randomly generated numbers (e.g., ranging from 1 to 20). During a one-minute timed assessment, the student reads aloud as many numbers as possible while the examiner records any Number Identification errors.

Number Identification: Preparation
The following materials are needed to administer Number Identification (NID) Early Math CBM probes:

- Student and examiner copies of a NID assessment probe. (**Note:** Customized NID probes can be created conveniently and at no cost using Numberfly, a web-based application. Visit Numberfly at *http://www.interventioncentral.org/php/numberfly/numberfly.php*).

- A pencil, pen, or marker

- A stopwatch

Number Identification: Directions for Administration

11. The examiner sits with the student in a quiet area without distractions. The examiner sits at a table across from the student.

12. The examiner says to the student:

    "The sheet on your desk has rows of numbers."

    "When I say, 'start,' begin reading the numbers aloud. Start at the top of this page and read across the page [demonstrate by pointing]. *Try to read each number. When you come to the end of a row, go to the next row. Are there any questions?* [Pause] *Start. "*

13. The examiner begins the stopwatch when the student reads the first number aloud. If the student hesitates on a number for 3 seconds or longer, the examiner says, *"Go to the next one."* (If necessary, the examiner points to the next number as a student prompt.)

14. The examiner marks each Number Identification error by marking a slash (/) through the incorrectly read number on the examiner form.

15. At the end of one minute, the examiner says, *"Stop"* and writes in a right-bracket symbol ( ]) on the examiner form from the point in the number series that the student had reached when the time expired. The examiner then collects the student Number Identification sheet.

Number Identification: Scoring Guidelines

Correct NID responses include:

- Numbers read correctly
- Numbers read incorrectly but corrected by the student within 3 seconds

Incorrect NID responses include:

- Numbers read incorrectly
- Numbers read correctly after hesitations of 3 seconds or longer
- Numbers skipped by the student

To calculate a Number Identification fluency score, the examiner:

1. counts up all numbers that the student attempted to read aloud and
2. subtracts the number of errors from the total of numbers attempted.
3. The resulting figure is the number of correct numbers identified.(NID fluency score).

Number Identification Probes as Part of a Response to Intervention Model

- Universal Screening: To proactively identify children who may have deficiencies in development of foundation math concepts, or 'number sense' (Berch, 2005), schools may choose to screen all kindergarten and first grade students using Number Identification probes. Those screenings would take place in fall, winter, and spring. Students who fall below the 'cutpoint' of the 35th percentile (e.g., Jordan & Hanich, 2003).of the grade norms on the NID task would be identified as having moderate deficiencies and given additional interventions to build their 'number sense' skills.

- Tier I (Classroom-Based) Interventions: Teachers can create Number Identification probes and use them independently to track the progress of students who show modest delays in their math foundation skills.

- Tier II (Individualized) Interventions. Students with more extreme academic delays may be referred to a school-based problem-solving team, which will develop more intensive, specialized interventions to target the student's academic deficits (Wright, 2007). Number Identification probes can be used as one formative measure to track student progress with Tier II interventions to build foundation math skills.

Number identification: Measurement Statistics

Test-Retest Reliability Correlations for Number Identification Probes		
Time Span	*Correlation*	*Reference*
13-week interval	0.85	Clarke & Shinn (2004)
26-week interval	0.76	Clarke & Shinn (2004)

Predictive Validity Correlations for Number Identification Probes		
Predictive Validity Measure	*Correlation*	*Reference*
Curriculum-Based Measurement Math Computation Fluency Probes: Grade 1 Addition & Subtraction (Fall Administration of MN Probe and Spring Administration of Math Computation Probe)	0.60	Clarke & Shinn (2004)
Woodcock-Johnson Tests of Achievement: Applied Problems subtest (Fall Administration of NID Probe and Spring Administration of WJ-ACH subtest)	0.72	Clarke & Shinn (2004)
Number Knowledge Test	0.58	Chard, Clarke, Baker, Otterstedt, Braun & Katz.(2005) cited in Gersten, Jordan & Flojo (2005)

References

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment For Effective Intervention, 30*(2), 3-14.

Clarke, B., & Shinn, M. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*, 234–248.

Gersten, R., Jordan, N.C., & Flojo, J.R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38,* 293-304.

Jordan, N. C. & Hanich, L. B. (2003). Characteristics of children with moderate mathematics deficiencies: A longitudinal perspective. *Learning Disabilities Research and Practice, 18*(4), 213-221.

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38,* 333-339..

Wright, J. (2007). *The RTI toolkit: A practical guide for schools*. Port Chester, NY: National Professional Resources, Inc.

Curriculum-Based Measurement: Early Mathematics Fluency Norms (Chard, Clarke, Baker, Otterstedt, Braun, & Katz, 2005)*

Early Math Fluency measures assess the strength of a student's 'number sense' (Chard, et al., 2005) and are good predictors of mathematical readiness at Kindergarten and Grade 1. Early Math Fluency measures include Quantity Discrimination, Missing Number, and Number Identification. All Early Math Fluency assessments have an administration time of 1 minute.

Quantity Discrimination (QD): 1 Minute: The student is presented with pairs of numbers randomly sampled from 1-20 and must identify the larger number in each pair.

Grade	Fall QD	Fall:+/-1 SD (≈16th%ile to 84th%ile)	Winter QD	Winter: +/-1 SD (≈16th%ile to 84th%ile)	Spring QD	Spring: +/-1 SD (≈16th%ile to 84th%ile)	Weekly Growth
K	15	8↔22	20	8↔32	23	12↔34	0.25
1	23	16↔30	30	21↔39	37	28↔46	0.43

Missing Number (MN): 1 Minute: The student is presented with response items consisting of 3 sequential numbers with one of those numbers randomly left blank. (Each 3-number series is randomly generated from the pool of numbers 1-20.) The student attempts to name the missing number in each series.

Grade	Fall MN	Fall: +/-1 SD (≈16th%ile to 84th%ile)	Winter MN	Winter: +/-1 SD (≈16th%ile to 84th%ile)	Spring MN	Spring: +/-1 SD (≈16th%ile to 84th%ile)	Weekly Growth
K	3	0↔7	10	3↔17	14	5↔21	0.34
1	9	3↔15	17	11↔23	20	14↔26	0.34

Number Identification (NID): 1 Minute: The student is presented with a randomly generated series of numbers ranging from 1-20 and names as many of those numbers aloud as time allows.

Grade	Fall NID	Fall: +/-1 SD (≈16th%ile to 84th%ile)	Winter NID	Winter: +/-1 SD (≈16th%ile to 84th%ile)	Spring NID	Spring: +/-1 SD (≈16th%ile to 84th%ile)	Weekly Growth
K	14	0↔28	45	27↔63	56	38↔74	1.31
1	34	18↔50	53	36↔70	62	46↔78	0.87

Reference: Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment for Effective Intervention, 30*(3), 3-14.

*Reported Characteristics of Student Sample(s) Used to Compile These Norms: *Number of Students Assessed:* Kindergarten: 168; Grade 1: 207/*Geographical Location:* Pacific Northwest: Sample drawn from 7 elementary schools in one district of 5500 students/ *Socioeconomic Status:* Students qualifying for free and reduced lunch: Range of 27% to 69% across 7 participating schools/*Ethnicity:* District population: 13% minorities/*ELLs:* District Population: 4% English Language Learners

Where to Find Materials: Schools can create their own CBM Early Math Fluency assessment materials at no cost, using NumberFly, a free online application:*http://www.interventioncentral.org/tools/early-math-fluency-generator*
This program generates printable student and examiner assessment sheets for CBM Quantity Discrimination, Missing Number, and Number Identification. From this site, the user can also download guidelines for administering and scoring these Early Math Fluency measures.